Time in System and Length of Stay Statistics: Interactions Among and Interpretations of System Flow Measures

Howard N. Snyder
National Center for Juvenile Justice
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A juvenile is arrested, held in police lockup for seven hours, and then transferred to the locally-operated detention center where he is held until his adjudicatory hearing. At this hearing, the youth is adjudicated delinquent and transferred to the custody of the state’s department of juvenile corrections. However, no beds are available in the state’s diagnostic and evaluation facility, so he stays in the local detention center for an additional five days awaiting state placement. When the state bed is available, he is transferred to the diagnostic and evaluation facility for 30 days and then transferred to the state training school for 9 months. At the end of the 9 months, he is placed on aftercare and transferred to a halfway house run by the local juvenile court. After a month at the halfway house, he is released on parole. He violates the conditions of parole once in the next 11 months and for this violation he spends a weekend in the local detention center. After a year on aftercare, he is released and his case is terminated. How many custody events did this youth have? How many times was this youth admitted into custody? What was his length, or his lengths, of stay in custody?

The most useful answers to these questions depend on who asks them and why they need the information. To the youth or to a state juvenile justice planner, the experience may be a single admission with the length of stay being the sum of all the days that the youth spend away from home. To the local detention center director, it may be two admissions with two separate lengths of stay. To the local court administrator, there may be three admissions with a length of stay equal to the detention time plus the
time in the halfway house the local court controls. To the state department of juvenile
corrections, there may be one admission with a total length of stay equal to the time in the
diagnostic and evaluation facility and the state training school. The concern of the state
official that monitors compliance with the Office of Juvenile Justice and Delinquency
Prevention’s mandates will be the time in lockup if this time was over the maximum
number of hours permitted by the JJDP Act.

When questions address the custody experience of more than one juvenile (e.g.,
juveniles in a specific facility), the descriptions of system flow (i.e., admissions, releases,
stock populations, and lengths of stay) become more complex. With each resident’s
length of stay in each facility and each entry and exit date, many population flow
measures can be constructed. Each measure captures one aspect of a multi-dimensional
phenomenon. These aspects are inter-related with a change in one causing at times
complex changes in the others. Even though they are hard to precisely define or
operationalize, these measures have become common in the language of juvenile justice.
Some of the more common measures of system flow are:

- **Daily Admissions (Admissions Population or Entry Cohort)** — the
  number of youth entering a facility or a system component (i.e., state
custody) in a particular time period.

- **Average Sentence of an Admission Cohort**— the average number of days
  that will be spent in the facility or system component by youth who
  entered the facility or system component in a defined time period.

- **Stock (or Daily) Population** — the number of youth in a facility or a
  system component on a particular day.

- **Average Stock Population** — the average of the daily populations over a
  defined time period.
• Average Time in System of the Stock Population — the average number of days that youth in a facility or system component has been in the facility or system component on the reference date.

• Daily Releases (Release Population or Exit Cohort) — the number of youth leaving a facility or a system component in a particular time period.

• Average Length of Stay of a Release Cohort— the average number of days spent in the facility or system component by youth who were released from the facility or system component in a defined time period.

Decision-makers must assess the differential information utility of available measures. Each measure has its appropriate uses, its strengths and weaknesses. Used in isolation, each measure gives a picture of juvenile custody or changes in system flow. Used together, the contradiction in the trends of different measures can present a confusing picture due to the complex interrelationships among the various measures.

The measures are easier to interpret (and the data to construct them easier to collect) when they describe a single facility or single system component than if they span a series of custody transactions. For example, for a training school director, monitoring the daily population gives the exact information needed to determine the number of meals that must be prepared by his food service vendor. With length of stay measures for an exit cohort, the changing character of the facility’s population could be monitored and some valuable information provided to help customize educational, medical and treatment programs for the youth who spent short or long stays in the facility. With stock population measures from facilities across the country, the federal government could monitor changes in percentage of available beds occupied by facility type and determine the relative need for infrastructure development. With similar information, private vendors could determine the needs of the market and move to fill custody gaps or to reduce surplus services. With facility-specific length of stay measures, a state juvenile
justice planning agency could compare the state’s use of custody to that of neighboring or similar jurisdictions.

However, when the focus of the population flow measures is expanded to summarize a series of custody transactions (e.g., youths’ total lockup, detention, state custody, and aftercare experiences), the interpretation and comparison of population flow problems grow. Data capturing the date of a youth’s admission to lockup and the date of his release from aftercare tells little about the custody experience or about the justice system’s response to this youth. To understand the complete experience, a youth’s facility-based flow data must be linked so that multivariate descriptions can be prepared and combined with the experiences of other youth. This larger focus increases the complexity of the interpretations of and the relationships among the various flow measures.

The goal of the current work is to demonstrate and explore the interrelationships among the various measures of system flow. For this conceptual work, the custody experience will be limited to the study of facility-based population flow measures. This work will attempt to (1) distinguish the characteristics of measures that quantify the facility population and length of stay measures and (2) describe the interrelationships among these measures and their trends. In this work it will be assumed that length of stay means the time between a youth’s date of entry into a facility and his date of release from the facility. If he returns to the same facility a week, a month or a year later, a new time period is initiated. Actual stays in a facility may be more complex, with temporary releases for court appearances, medical services, or escapes; but for this conceptual work, these complexities will be ignored. The interpretation problems associated with changing
flow measures are tied to their complex interrelationships and the necessity at times to use one measure as a surrogate for a more appropriate, but available, measure. The next chapter will present examples of the relationships among the various system flow measures and the different pictures they present under different flow dynamics.
Chapter 2
The Interrelationships among Population Flow Measures

In the simplest of all custody scenarios, a new facility opens and admits the same number of juveniles each day; each youth then stays in the facility for the same number of days before being released. In this flow scenario, after a period of time equal the fixed length of stay, the system reaches a point of equilibrium and the other system flow measures/attributes (i.e., size of the daily population, time in the facility for the stock population, daily number of releases, and lengths of stay in custody of youth released) remain at a constant value. For example, assume a facility (a large facility) admits 100 youth per day for 1000 days and each youth stay is the facility for 50 days (see Figure 1). Under this condition, the stock population increases for each of the first 50 days and then remains stable at 5000. The size of the release population is zero for the first 50 days and then remains constant thereafter at 100 releases per day. The average time in facility of the stock population increases for the first 50 days and then remains constant at 25.5 days. At equilibrium the distribution of the times in system for the stock population on any day would be flat, with 100 youth in the facility for one day, 100 youth in the facility for two days, etc. No youth leaves the facility for the first 50 days; afterwards, all youth who leave the facility have been there for 50 days. Consequently, with a consistent daily intake and a fixed length of stay (i.e., a constant value), the system (the facility) functions in a state of equilibrium once the size of the stock population reaches stability. The general equations for this family of flow parameters are presented in Table 1.
Of special interest to the present discussion is the relationship between the
distribution of lengths of stay of the release population and the distribution of times in
system of the stock population. Under the set of flow conditions presented in the above
example, the daily distribution of the times in system for the stock populations would
also be invariant at equilibrium, with equal height bars across the range of time in system
values. At equilibrium the Length of Stay distribution for youth released from custody in
a specified time period is a single bar at the fixed length of stay.

Effect of a Change in Length of Stay on Flow Parameters

A change to any of the system flow characteristics would be reflected in changes
in some or all of the other measures. Consider what would happen to the flow parameters
if daily admissions were to remain constant but the length of stay of the admission
population were to instantly double on a single day (see Figure 2). More specifically,
assume that on a single day the length of stay of all new admissions were to increase
from 50 to 100 days. Over the resultant transition period, the size of the stock population
would hold constant for 50 days and then gradually increase to a new equilibrium point
after 100 days. The size of the release cohort would hold constant until all youth who
had received a sentence of 50 days had been released from the system. Then for 50 days,
no youth would be released from the facility. Later, on the day the first youth who
entered the facility with the longer length of stay left the facility, the size of the release
population would return to its previous level. The average length of stay of the exit
population would jump to 100 days on this day. Over this transition period the average
time in system of the stock population would remain constant for the first 50 days and
then to increase for an additional 50 days, eventually reaching a new equilibrium point at
one-half the population’s new length of stay. Over the transition period the daily distributions of length of stay would either be a single bar at 50 days, an empty distribution during the period when no one is being released, or a single bar at 100 days. During this period the daily distributions of the stock population’s times in system would vary:

- from a flat distribution with equal numbers of youth at each time in system from one day to 50 days;
- to a series of distributions which change daily, adding a bar to the next higher time in system value between 51 and 100;
- to a new flat distribution in which every time in system value from one day to 100 days has the same number of residences.

If the sentence of the intake population were to revert abruptly back to the 50-day level, complimentary changes would occur to those observed following an abrupt increase.

**Effect of a Change in Admissions on Flow Parameters**

Now, consider what would occur if, after an equilibrium state was reached, the admissions were to increase (or decrease) on a single day and remained at this level for a long period of time (see Figure 3). As the figure shows, the change in admissions influences the distributions of the other flow parameters. Specifically, assume that after a period of equilibrium, the number of youth admitted to a facility were to increase by 50% on a single day and remained at this higher level for an extended period of time. On the day the increase occurred, the size of the stock population would begin to gradually increase until it reached a new equilibrium point. At this new equilibrium point the stock population would be 50% greater than the previous level. The size of the release population would abruptly increase by 50% fifty days (the constant length of stay) after
the date of the increase in admissions. In this scenario the average time in facility of the stock population would begin to decrease on the day of the population increase (with the higher number of new admissions pulling down the mean time in system). The decline in the mean time in system would continue for 25 days (one-half the constant length of stay) and then over the next 25 days return to its level prior to the admissions increase. Even with an increase in the intake population, the average length of stay of the release population would remain constant at 50 days. Fifty days after the admissions increase, in a state of constant intake flow and constant length of stay, the system would function once again in a state of equilibrium. (As the figure shows, an abrupt decline in the intake population would have the opposite effect on the size of the stock population, the average time in system of the stock population, and the size of the release cohort.)

Following an abrupt increase in the intake population, the distribution of lengths of stay for youth released from custody in a specified time period would still be a single bar at the 50-day point, although the height of the daily distribution would jump to a new level 50 days after the date of the admissions increase. The daily distributions of the times in system for a stock population would respond to the admissions increase with a wave of taller bars moving across the x-axis between the date of the admissions increase and 50 days later until a new equilibrium point is reached. Correspondingly, the daily distributions for times in system for a stock population would respond to the admissions decrease with a wave of shorter bars moving across the x-axis between the date of the admissions decrease and 50 days later followed by a period of system equilibrium.
Systems with Variable Lengths of Stay

With this preparation in reading families of flow parameter distributions, consider what would occur in systems if the daily intake flow remained constant but the lengths of stay of youth in the admissions cohort varied.

*Flat Length of Stay Distribution* — Instead of a single, fixed length of stay as in the previous examples, assume that sentences ranged from one day to a maximum number of days with an equal number of youth in each daily admission cohort receiving a sentence of one day, two days, etc. This flat distribution of sentences coupled with constant system intake results in its own family flow parameter distributions (see Figure 4).

At equilibrium the distribution of sentences in the admission cohorts is reflected in the distribution of lengths of stay of the exiting cohort. At equilibrium the *Length of Stay* distribution, however, yields a positively skewed, triangular *Time in System* distribution. The size of the exit cohort is equal to the size of the admissions cohort when the system is in a state of equilibrium. The size of the stock population at equilibrium is the sum of the magnitudes of the bars in the *Time in System* distribution. This sum is a function of both the size of the daily admissions cohort and the maximum length of stay in the triangular distribution of times in system. The average time in system and the average length of stay for this system are weighted averages of the *Time in System* and *Length of Stay* distributions, respectively. Physically, the average value of these two distributions is the point on the x-axis where a two-dimensional plane with the shape of the distribution would balance, or the x-value of distribution’s center of gravity. For the
Length of Stay distribution, this point is at the distribution’s midpoint on the x-axis. For the Time in System distribution, the center of gravity is to the left of the distribution’s midpoint on the x-axis. The general equations for these flow parameters are presented in Table 1.

Abrupt changes in the size of the admissions cohort are registered in the family flow parameters in patterns similar to those discussed previously. With the increase in admissions, the size of the stock population begins to increase immediately; though delayed, the size of the release population also increases; the average time in system of the stock population declines temporarily; as does the average length of stay of the release cohort. The distributions of Time in System and Length of Stay also respond to the abrupt changes in admissions, with the distributions morphing themselves from one state of equilibrium to another.

Negatively Skewed Triangular Length of Stay Distribution — In a different custody system the modal sentence is the maximum length of stay, the fewest youth are sentenced to just one day, and the difference between the number of youth sentence to N+1 days and N days is a constant. This negatively skewed triangular length of stay distribution coupled with constant system intake has in its own family flow parameter distributions (see Figure 5).

At equilibrium the distribution of sentences in the admission cohorts is reflected in the distribution of lengths of stay of the exiting cohort and the size of the exit cohort is equal to the size of the admissions cohort. At equilibrium the negatively skewed, triangular Length of Stay distribution yields a convex Time in System distribution. This
convex distribution is characterized by the property that the difference between the number of youth in the system N days and N+1 days increases with the number of days in system. The size of the stock population at equilibrium is the sum of the magnitudes of the bars in the *Time in System* distribution, which is a finite sum of a function involving both the size of the daily admissions cohort and the maximum sentence. The average time in system and the average length of stay for this system are weighted averages of the *Time in System* and *Length of Stay* distributions, respectively. For the *Length of Stay* distribution, this point is between at the distribution’s midpoint on the x-axis and its maximum value. For the *Time in System* distribution, the average value (or the distribution’s center of gravity) is to the left of the distribution’s midpoint on the x-axis.

The general equations for these flow parameters are presented in Table 1. The family flow parameters respond to abrupt changes in the size of the admissions cohort in patterns following those noted previously. The distributions of *Time in System* and *Length of Stay* also respond to the abrupt changes in admissions, with the distributions morphing themselves from one state of equilibrium to another.

*Positively Skewed Triangular Length of Stay Distribution* — Assume a custody system with a modal sentence is one day, with fewer youth with a sentence of two days, still fewer youth with a sentence of three days, and so on until the fewest youth stayed the maximum number of days. Further assume that the difference between the number of youth sentence to N days and N+1 days is a constant. This is a positively skewed triangular length of stay distribution. This distribution of lengths of stay coupled with a constant system intake results in its own family flow parameter distributions (see Figure 6).
Once again, at equilibrium the distribution of sentences in the admission cohorts is reflected in the distribution of lengths of stay of the exiting cohort and the size of the exit cohort is equal to the size of the admissions cohort. At equilibrium the positively skewed, triangular *Length of Stay* distribution, however, yields a concave *Time in System* distribution. This concave distribution is characterized by the property that the difference between the number of youth in the system N days and N+1 days decreases with the number of days in system. The size of the stock population at equilibrium is the sum of the magnitudes of the bars in the *Time in System* distribution, which is a finite sum of a function involving both the size of the daily admissions cohort and the maximum sentence. The average time in system and the average length of stay for this system are weighted averages of the *Time in System* and *Length of Stay* distributions, respectively. For the *Length of Stay* distribution, this point is on the x-axis between at the distribution’s midpoint and its minimum value, and has a smaller value than that of a comparable negatively skewed *Length of Stay* system with similar minimum and maximum values. For the *Time in System* distribution, the average value (or the distribution’s center of gravity) is to the left of the distribution’s midpoint on the x-axis, and less than the average time in system for a negatively skewed *Length of Stay* system with similar minimum and maximum values. The general equations for these flow parameters are presented in Table 1. The family of flow parameters responds to abrupt changes in the size of the admissions cohort in patterns following those noted previously. During transition from one equilibrium state to another, the distributions of times in system and lengths of stay morph from one fixed distribution to another.
Symmetrical Triangular Length of Stay Distribution — Building on this set of prototypical systems, imagine a custody system where the modal sentence is half of the maximum sentence and the number of youth sentenced to more or less than the modal sentence declines consistently so that the fewest number of youth stay either one day or the maximum number of days. This system has a symmetrical triangular Length of Stay distribution, a rough approximation of a normal distribution. Coupled with constant system intake, this system has family flow parameter distributions that are combinations of the preceding models (see Figure 7).

At equilibrium the distribution of sentences in the admission cohorts is reflected in the distribution of lengths of stay of the exiting cohort. At equilibrium this combination of a positively and negatively skewed Length of Stay distribution yields a Time in System distribution that is a combination of their corresponding Time in System distributions. The first part of the Time in System distribution is convex and the latter part is concave. The size of the exit cohort is equal to the size of the admissions cohort when the system is in a state of equilibrium. The size of the stock population at equilibrium is the sum of the magnitudes of the bars in the Time in System distribution. This sum is the combination of two separate functions, one corresponding to a negatively-skewed, triangular distribution of length of stay that ranges from the minimum sentence to half of the maximum sentence and one corresponding to a positively-skewed, triangular distribution of length of stay ranging from the half of the maximum sentence to the maximum sentence. Abrupt changes in the size of the admissions cohort are registered in the distributions of Time in System and Length of Stay, with these
distributions stepping through complex transitional patterns until the system stabilizes at a new state of equilibrium.

**The Interrelationship of Time in System and Length of Stay Distributions**

This last system scenario highlights a general property of *Time in System* distributions. Under constant flow and sentencing conditions, *Time in System* distributions can be decomposed into a set of simpler distributions, each tied to a simple *Length of Stay* distribution. *Time in System* and the *Length of Stay* distributions are, therefore, linked together, but the conversion from one to another is not a simple task.

For example, once equilibrium is reached, a system with a singular length of stay (i.e., all youth stay the same number of days) and a constant number of daily admissions produces a flat *Time in System* distribution from one day through the day equal to the singular length of stay. That is, if one facility has a constant intake flow of 10 youth per day and a constant length of stay of 25 days, the *Time in System* distribution of the stock population on any day (once equilibrium is reached) is a series of equal height bars for each day between the Day 1 and Day 25. The height of each bar is 10, equal to the daily admission population. If another facility has a constant intake flow of 20 youth per day and a constant length of stay of 75 days, the *Time in System* distribution on any day (once equilibrium is reached) is also a set of equal height bars for each day between Day 1 and Day 75 with height equal to 20 youth, the daily admission population. If, now, these two facilities were combined into a single facility, their corresponding distributions are added together to form a new *Time in System* distribution. The daily exit cohort’s *Length of Stay* distribution for the new facility would be bi-modal, with a bar of height 10 at 25
days and a bar of height 20 at 75 days. The *Time in System* distribution for the new facility would not be flat. The distribution would have a set of larger bars for the *Time in System* days that are common to both lengths of stay (i.e., Day 1 through Day 25 would have bars with heights of 30) and a set of bars that are equal in height to the original distribution for days that are unique to one of the two original distributions (i.e., Day 26 through Day 75 with heights of 20 youth).

This same process could be expanded to capture three, four, or even two hundred different singular *Length of Stay* distributions. At equilibrium with constant daily admissions, each *Length of Stay* distribution would add its own contribution to the heights of the bars in the joint *Time in System* distribution, a contribution equal to the admissions population for its distribution. Therefore, knowing the constant admission population for each *Length of Stay* distribution, the joint *Time in System* distribution could be easily constructed. However, to move in the opposite direction, from a single *Time in System* distribution to the composite *Length of Stay* distribution, is not as easy. There are likely to be many sets of *Length of Stay* distributions that could yield a single *Time in System* distribution. Any single solution would not be unique — even with the assumption of constant daily admissions for each subcomponent of the overall *Length of Stay* distribution. Then complicate the problem of generating a single *Length of Stay* distribution from a joint *Time in System* distribution for multiple facilities (as would be the case when working with State or national data), the relationship between the *Time in System* and *Length of Stay* distributions would be far from unique.

The capability to predict one distribution from the other becomes clearly more difficult when the fixed lengths of stay and/or daily admissions populations vary.
randomly around a defined probability distribution. For example, assume a system in which the probability of any individual length of stay were equal throughout the range of possible lengths of stay, from one day to the maximum number of days. Over an infinite number of sample systems, this system would display a flat *Length of Stay* distribution and, correspondingly, a positively skewed, triangular *Time in System* distribution ranging from one day to the maximum sentence, represented previously in Figure 4. However, for any specific sample system (i.e., a system with lengths of stay generated using a probability model), the distinguishable flat *Length of Stay* and positively skewed triangular *Time in System* distributions are distorted, loosing much of their recognizable characteristics even with this relatively minor variance from the fixed flow characteristics (see Figure 8). The sharp and trademark shapes of other paired *Length of Stay* and *Time in System* distributions also begin to dissolve when a small amount of instability is added to the key system characteristic of length of stay (see Figures 9 and 10).

The problem of linking *Time in System* and *Length of Stay* distributions magnifies as these idealized systems move closer and closer to reality. As can be seen in Figures 8, 9, and 10, a single change in daily admissions has a major (though transitory) effect on the shapes of these distributions. Now add the reality that for any facility the admission, stock and exit populations are relatively small and normal sample variations would generate sizeable variances in population characteristics, the task of confidentially linking

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1 For this example, and unlike in the earlier system flow scenarios, the length of stay of each youth admitted to the facility was determined probabilistically. Each possible length of stay was linked to a part of the range of numbers from 1 to 1000. For the flat distribution ranging from 1 to 50 days, the numbers assigned to a length of stay of one day were 1 through 20, for two days 21 through 40, and so on. As each youth was admitted, a random number was generated and that youth was assigned the corresponding length of stay. With this generator, each length of stay was equally probable. However, the number of youth receiving any specific length of stay varied randomly around the expected (i.e., equal) mean value.
a Time in System distribution with a Length of Stay distribution becomes more problematic. If we were then to assume that some or all of the facilities that generated a Time in System distributions were not in equilibrium, (i.e., systems with daily populations and lengths of stay varying constantly), the task of confidentially constructing a single Length of Stay distribution from a facility’s Time in System distribution becomes an apparently unsolvable problem.

A Consideration of Actual Distributions

The 1997 Census of Juveniles in Residential Placement provides for the first time a detailed delineation of the Time in System distribution youth in custody in the United States. These nearly 106,000 youth were held in many different types and sizes of facilities on a Wednesday in October 1997. On that census date, they were also in a range of legal statuses. Even when the census population is divided into three groups (i.e., those youth who were held in detention awaiting adjudication or placement, those youth who had been committed to the facility, and those youth who were awaiting criminal court processing), their Time in System distributions all appear to have similar shapes (see Figures 11, 12, and 13).

An obvious characteristic of the distributions is the range of times in systems. Ninety-five percent of youth in the detention distribution have been in custody for 4 months or less, while a similar proportion of youth awaiting criminal court processing had been in custody 12 months or less. The time in system of committed had by far the large range, with 95% in custody 20 months or less.
Another characteristic of note is the similarities in their positively skewed, concave distributions. If a state of equilibrium is assumed, and based on earlier presentations, these distributions suggest that their Length of Stay distributions are themselves positively skewed. The relative degree of skewness, however, is worth attention. The detention and the criminal processing distributions drop more sharply than the committed distribution. They also level off into a flatter distribution than does the commitment distribution. Both of these characteristics suggest that the detention and criminal processing distribution contain a subset of youth who stay in the facility for a relatively short period of time and a subset of youth who stay in the facility for a lengthy period. In contrast, the Time in System distribution for committed declines more gradually, implying that their Length of Stay distribution is more evenly spread across the range of lengths of stay.

Finally, a clear characteristic of each distribution is the short-term cyclical fluctuations. These fluctuations cycle over a 7-day period. These weekly variations indicate that youth are far less likely to be admitted to a facility on Saturdays and Sundays than other days of the week. For committed youth and youth awaiting criminal court processing, this oscillation is probably tied to the fact that the courts placing them in the facilities sit on weekdays. The reason underlying the pattern for detained youth is less clear. It may be due to the fact that in some jurisdictions the court must order the youth to detention. However antidotal information indicates that law enforcement may be less willing to bring youth to detention centers on weekends, given the amount of time they must allot to the processing.
Final Thoughts

Most of this discussion has focused on \textit{Length of Stay} and \textit{Time in System} distributions. The mean \textit{Length of Stay} and \textit{Time in System} statistic have been characterized as the x-axis values of the center of gravity of each distribution. This value of this point is sensitive to small changes in the shape of the distribution and is greatly influence by outliers. Correspondingly, substantial changes in these distributions may have no effect on mean values if the changes happen to compensate for each other.

Finally, from observing CJRP data, the \textit{Length of Stay} and \textit{Time in System} distributions are not normally distributed. Therefore, testing differences in mean values from one wave of data collection to another or between one subpopulation and another is not a straightforward exercise and any findings would be difficult to interpret. For example, big changes in a distribution could yield no change in mean values. In all, the unique conversion of a \textit{Time in System} distribution to a \textit{Length of Stay} distribution is likely to be an unsolvable problem, even under equilibrium conditions. And without the equilibrium constraints, the bond between the two distributions degenerates even more.
Figure 1
Distributions of Flow Parameters Under Conditions of Constant Number of Admissions and Fixed Length of Stay
Figure 2
Distributions of Flow Parameters Under a Condition of Fixed Daily Admissions

- **Daily Admissions**
  - Days Since Opening vs. Juveniles
  - Days Since Opening vs. Sentences at Admission

- **Size of Stock Population**
  - Days Since Opening vs. Juveniles

- **Size of Release Cohort**
  - Days Since Opening vs. Juveniles

- **Average Time in System of Stock Population**
  - Days Since Opening vs. Days

- **Average Length of Stay of Release Cohort**
  - Days Since Opening vs. Days
Figure 2 (continued)
Distributions of Flow Parameters Under a Condition of Fixed Daily Admissions

Times in System During System Equilibrium

Lengths of Stay During System Equilibrium

Times in System During Population Increase

Lengths of Stay During Population Increase

Times in System During Population Decrease

Lengths of Stay During Population Decrease
Figure 3
Distributions of Flow Parameters Under a Condition of a Fixed Length of Stay

Admissions Characteristics

Distribution of Sentences at Admission

Size of Stock Population

Size of Release Cohort

Average Time in System of Stock Population

Average Length of Stay of Release Cohort
Figure 3 (continued)
Distributions of Flow Parameters Under a Condition of a Fixed Length of Stay

Distribution of Time in System During System Equilibrium

Distribution of Length of Stay During System Equilibrium

Distribution of Time in System During Population Increase

Distribution of Length of Stay Once Equilibrium Reached After Population Increase

Distribution of Time in System During Population Decrease

Distribution of Length of Stay Once Equilibrium Reached After Population Decrease
Figure 4
Distributions of Flow Parameters Under a Condition of a Flat Length of Stay Distribution

- **Daily Admissions**
- **Sentences at Admission**
- **Size of Stock Population**
- **Size of Release Cohort**
- **Average Time in System of Stock Population**
- **Average Length of Stay of Release Cohort**
Figure 4 (continued)
Distributions of Flow Parameters Under a Condition of
a Flat Length of Stay Distribution

Times in System During System Equilibrium

Lengths of Stay During System Equilibrium

Times in System During Population Increase

Lengths of Stay During Population Increase

Times in System During Population Decrease

Lengths of Stay During Population Decrease

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Figure 5
Distributions of Flow Parameters Under a Condition of a Continuously Increasing Length of Stay Distribution

Daily Admissions

Sentences at Admission

Size of Stock Population

Size of Release Cohort

Average Time in System of Stock Population

Average Length of Stay of Release Cohort

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Figure 5 (continued)
Distributions of Flow Parameters Under a Condition of a Continuously Increasing Length of Stay Distribution

Times in System During System Equilibrium

Lengths of Stay During System Equilibrium

Times in System During Population Increase

Lengths of Stay During Population Increase

Times in System During Population Decrease

Lengths of Stay During Population Decrease
Figure 6
Distributions of Flow Parameters Under a Condition of a Continuously Decreasing Length of Stay Distribution

- **Daily Admissions**
- **Sentences at Admission**
- **Size of Stock Population**
- **Size of Release Cohort**
- **Average Time in System of Stock Population**
- **Average Length of Stay of Release Cohort**
Figure 6 (continued)

Distributions of Flow Parameters Under a Condition of a Continuously Decreasing Length of Stay Distribution

Times in System During System Equilibrium

Lengths of Stay During System Equilibrium

Times in System During Population Increase

Lengths of Stay During Population Increase

Times in System During Population Decrease

Lengths of Stay During Population Decrease
Figure 7
Distributions of Flow Parameters Under a Condition of a Triangular Length of Stay Distribution

Daily Admissions

Sentences at Admission

Size of Stock Population

Size of Release Cohort

Average Time in System of Stock Population

Average Length of Stay of Release Cohort
Figure 7 (continued)
Distributions of Flow Parameters Under a Condition of a Triangular Length of Stay Distribution

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<th>Lengths of Stay During System Equilibrium</th>
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</thead>
<tbody>
<tr>
<td>Days in Facility on Day 200</td>
<td>Days in Facility on Day 215</td>
</tr>
<tr>
<td>Days in Facility on Day 326</td>
<td>Days in Facility on Day 341</td>
</tr>
<tr>
<td>Days in Facility on Day 626</td>
<td>Days in Facility on Day 641</td>
</tr>
</tbody>
</table>

Juveniles in Stock Population

Juveniles in 30-Day Release Cohort

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Figure 8
Distributions of Flow Parameters Under a Condition of a Flat Length of Stay Distribution with Random Variation

<table>
<thead>
<tr>
<th>Daily Admissions</th>
<th>Sentences at Admission</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Days Since Opening</strong></td>
<td><strong>Sentence in Days</strong></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>400</td>
<td>20</td>
</tr>
<tr>
<td>600</td>
<td>30</td>
</tr>
<tr>
<td>800</td>
<td>40</td>
</tr>
<tr>
<td>1000</td>
<td>50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Size of Stock Population</th>
<th>Size of Release Cohort</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Days Since Opening</strong></td>
<td><strong>Sentence in Days</strong></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>400</td>
<td>20</td>
</tr>
<tr>
<td>600</td>
<td>30</td>
</tr>
<tr>
<td>800</td>
<td>40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average Time in System of Stock Population</th>
<th>Average Length of Stay of Release Cohort</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Days Since Opening</strong></td>
<td><strong>Sentence in Days</strong></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>400</td>
<td>20</td>
</tr>
<tr>
<td>600</td>
<td>30</td>
</tr>
<tr>
<td>800</td>
<td>40</td>
</tr>
<tr>
<td>1000</td>
<td>50</td>
</tr>
</tbody>
</table>
Figure 8 (continued)
Distributions of Flow Parameters Under a Condition of a Flat Length of Stay Distribution with Random Variation

Times in System During System Equilibrium

Lengths of Stay During System Equilibrium

Times in System During Population Increase

Lengths of Stay During Population Increase

Times in System During Population Decrease

Lengths of Stay During Population Decrease

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Figure 9
Distributions of Flow Parameters Under a Condition of
a Continuously Decreasing Length of Stay Distribution with Random Variation

- Daily Admissions
- Sentences at Admission
- Size of Stock Population
- Size of Release Cohort
- Average Time in System of Stock Population
- Average Length of Stay of Release Cohort
Figure 9 (continued)
Distributions of Flow Parameters Under a Condition of a Continuously Decreasing of Stay Distribution with Random Variation

Times in System During System Equilibrium

Lengths of Stay During System Equilibrium

Times in System During Population Increase

Lengths of Stay During Population Increase

Times in System During Population Decrease

Lengths of Stay During Population Decrease
Figure 10
Distributions of Flow Parameters Under a Condition of a Triangular Length of Stay Distribution with Random Variation
Figure 10 (continued)
Distributions of Flow Parameters Under a Condition of a Triangular Length of Stay Distribution with Random Variation
Figure 11
Time in System of Detained Youth
in the 1997 Census of Juveniles in Residential Placement
Figure 12
Time in System of Committed Youth
in the 1997 Census of Juveniles in Residential Placement
Figure 13
Time in System of Youth Awaiting Criminal Court Processing
in the 1997 Census of Juveniles in Residential Placement
# Table 1

**Mathematical Descriptions of Flow Parameters at System Equilibrium for Selected System Prototypes**

<table>
<thead>
<tr>
<th>Flow Parameters</th>
<th>Condition 1: Constant Intake Flow Length of Stay = MAX (see Figure 1)</th>
<th>Condition 2: Constant Intake Flow Flat Length of Stay Distribution (see Figure 4)</th>
<th>Condition 3: Constant Intake Flow Negatively Skewed Length of Stay Distribution (see Figure 5)</th>
<th>Condition 4: Constant Intake Flow Positively Skewed Length of Stay Distribution (see Figure 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily Admissions (DA)</td>
<td>DA = Constant</td>
<td>DA = Constant</td>
<td>DA = Constant</td>
<td>DA = Constant</td>
</tr>
<tr>
<td>Distribution of Sentences: Number of Youth at Admission with Specific Sentence I</td>
<td>SENT$_I$ = 0, where I= 1 to (MAX – 1)</td>
<td>SENT$_I$ = [1/MAX] * DA, where I= 1 to Max Sentence</td>
<td>SENT$_I$ = [1/(MAX)*(MAX+1)/2] * I * DA, where I= 1 to Max Sentence</td>
<td>SENT$_I$ = (MAX +1- I) * [1/(MAX)*(MAX+1)/2] * DA, where I= 1 to Max Sentence</td>
</tr>
<tr>
<td>Size of Stock Population</td>
<td>DA * MAX</td>
<td>[(MAX+1)/2] * DA</td>
<td>∑ (I * I) [1/(MAX)*(MAX+1)/2] * DA, where I= 1 to Max Sentence</td>
<td>∑ (I * [1/(MAX)*(MAX+1)/2] * DA), where I= 1 to Max Sentence</td>
</tr>
<tr>
<td>Size of Release Cohort</td>
<td>DA</td>
<td>DA</td>
<td>DA</td>
<td>DA</td>
</tr>
<tr>
<td>Average Time in System</td>
<td>(MAX + 1)/2</td>
<td>1 + (MAX – 1)/3</td>
<td>0.3887 * MAX</td>
<td>0.2650 * MAX</td>
</tr>
<tr>
<td>Average Length of Stay</td>
<td>MAX</td>
<td>(MAX + 1)/2</td>
<td>MAX – [(MAX – 1)/3] + 1</td>
<td>1 + (MAX – 1)/3</td>
</tr>
<tr>
<td>Distribution of Times in System: Number of Youth with Time in System I</td>
<td>TIS$_I$ = DA, where I= 1 to Max Sentence</td>
<td>TIS$_I$ = (MAX +1- I) * [1/(MAX)*(MAX+1)/2] * DA, where I= 1 to Max Sentence</td>
<td>TIS$_I$ = DA,$ - \sum _{n=1}^{I} [1/(MAX)*(MAX+1)/2] * DA) where N = I, I and where I= 1 to Max Sentence</td>
<td>TIS$_I$ = [MAX +1 – I] * [1/(MAX)*(MAX+1)/2] * DA, where N = I, MAX</td>
</tr>
<tr>
<td>Distribution of Lengths of Stay: Number of Youth with Length of Stay I</td>
<td>LOS$_I$ = 0, where I= 1 to (MAX – 1)</td>
<td>LOS$_I$ = DA/MAX</td>
<td>LOS$_I$ = I * [1/(MAX)*(MAX+1)/2] * DA</td>
<td>LOS$_I$ = (MAX +1- I) * [1/(MAX)*(MAX+1)/2] * DA</td>
</tr>
</tbody>
</table>